Image Processing & Pattern

E1425

Lecture 4

Spatial Domain Linear Filtering

INSTRUCTOR

DR / AYMAN SOLIMAN

Contents

- Linear Shift-Invariant System
- Impulse Response
- Spatial Neighborhood
- Masks, Windows, Filters and the Impulse Responses
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➢ Linear Shift-Invariant System

- ❑ Linearity: “things can be added”
- ❑ Shift-invariance: “things do not change over space”

➢ Filtering with LSI System

- ❑ Spatial domain → Convolution
- ❑ Frequency domain → Multiplication (convolution theorem)

➢ Impulse Response

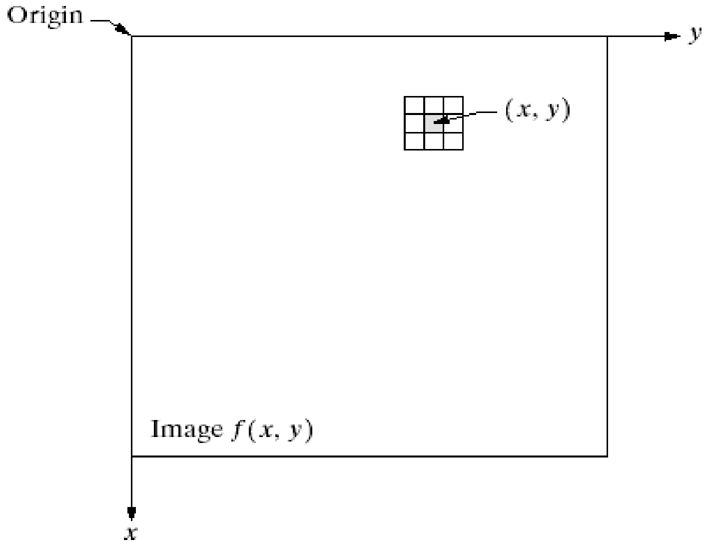
- The response of an LSI system to an impulse input

- KEY: An LSI system can be completely characterized by its impulse response

- Given the impulse response of an LSI system, together with the input to the system, the output is uniquely determined

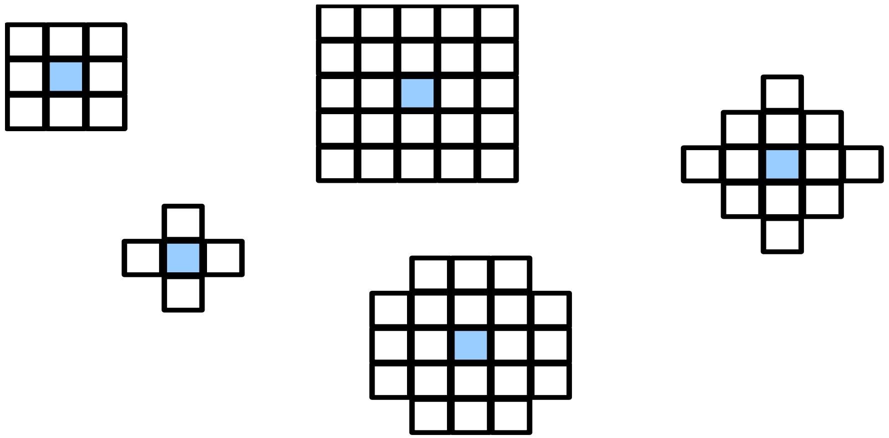
➢ Spatial Neighborhood

FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.



From [Gonzalez & Woods]

choices of neighborhood:



➢ Masks, Windows, Filters and the Impulse Responses

- ➢ Spatial LSI Filter:
  - ❑ impulse response constrained within a local neighborhood

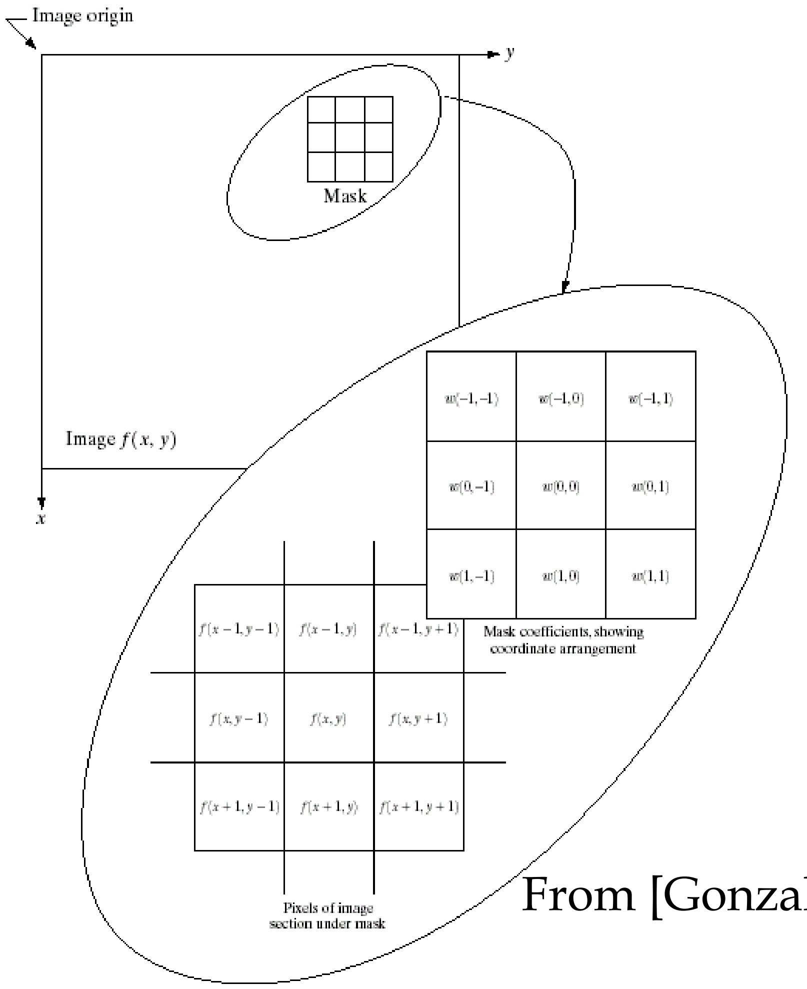


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

- ✓ “Filter”
- ✓ “Mask”
- ✓ “Window”
- ✓ ”Impulse Response”
- ✓ often used interchangeably for LSI

From [Gonzalez & Woods]

➢ 2D Convolution

$$x(m,n) \longrightarrow \boxed{h(m,n)} \longrightarrow y(m,n)$$

$$y(m,n) = \sum_{k,l=-\infty}^{\infty} h(k,l)x(m-k,n-l) = h(m,n) \otimes x(m,n)$$

$$y(m,n) = \sum_{k,l=-\infty}^{\infty} h(m-k,n-l)x(k,l) = x(m,n) \otimes h(m,n)$$

$h(m, n)$ → impulse response (spatial linear filter)

x(m, n) → input image

y(m, n) → output image

Applications

- Image Smoothing
- Image Enhancement
- Image Restoration
  - Image denoising
  - Image deblurring
- Edge Detection
- Filter Bank
  - Image transformation
  - Frequency analysis
- ……

➢ Image Smoothing: Average Filters

- **Average Filter**

$$h(m,n) = \frac{1}{N^2}\begin{bmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{bmatrix}$$

N: filter size

noisy

PSNR=20.2dB
noise std = 25

smoothed

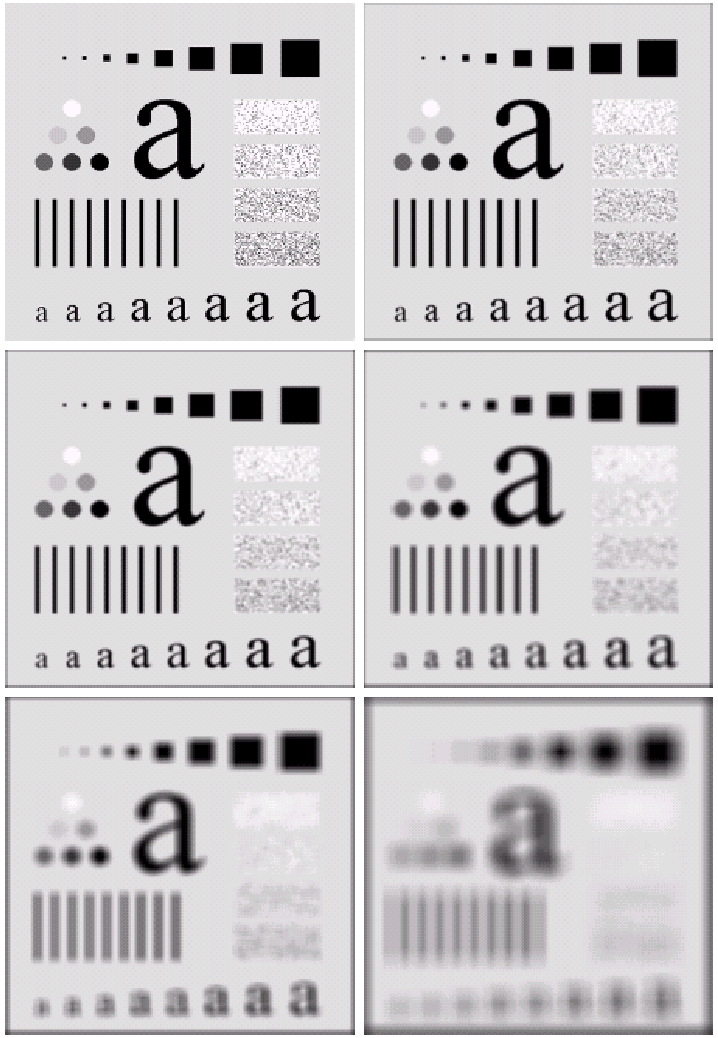
PSNR=23.8dB
3x3 window

smoothed

PSNR=22.0dB
5x5 window

➢ Image Smoothing: Average Filters

Original image size: 500x500
Average filtered images.
Filter sizes: 3, 5, 9, 15 and 35



➢ **Effects**

Smoothing noise
Blurring edges

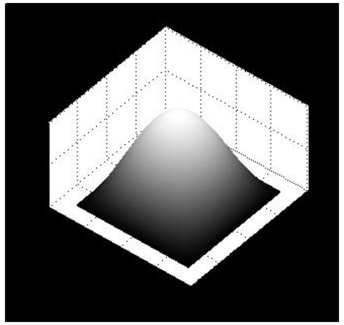
From [Gonzalez & Woods]

➢ Image Smoothing: Gaussian Filters

- **Gaussian Filter**

$$h(m,n) = \frac{1}{Z}\exp\left[-\frac{m^2+n^2}{2\sigma^2}\right]$$

$$-N \le m,n \le N$$



noisy	smoothed	smoothed
		
PSNR=20.2dB noise std = 25	PSNR=24.4dB σ=1	PSNR=22.8dB σ=1.5

➢ Image Smoothing Filter Example

- **Filter** $\frac{1}{6}$

0	1	0
1	2	1
0	1	0

- **Input image: A 4x4, 4 bits/pixel**

1	8	6	6
6	3	11	8
8	8	9	10
9	10	10	7

- **Preprocessing: Zero-padding**

1	8	6	6
6	3	11	8
8	8	9	10
9	10	10	7

→

0	0	0	0	0	0
0	1	8	6	6	0
0	6	3	11	8	0
0	8	8	9	10	0
0	9	10	10	7	0
0	0	0	0	0	0

➢ Image Smoothing Filter Example

- **Move mask across the zero-padded image**

$\frac{1}{6}$

0	1	0
1	2	1
0	1	0

0	0	0	0	0	0
0	1	8	6	6	0
0	6	3	11	8	0
0	8	8	9	10	0
0	9	10	10	7	0
0	0	0	0	0	0

- **Compute weighted sum**

- **Result:**

2.6	4.3	6.2	4.3
4.0	6.5	8.0	7.2
6.5	7.7	9.5	7.3
6.0	7.8	7.7	5.7

round →

3	4	6	4
4	7	8	7
7	8	10	7
6	8	8	6

➢ Sharpening Linear Filters

- **Laplacian** $$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$
  - Zero at smooth regions
  - Sensitive to image details

- **Discrete approximation of Laplacian:**

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

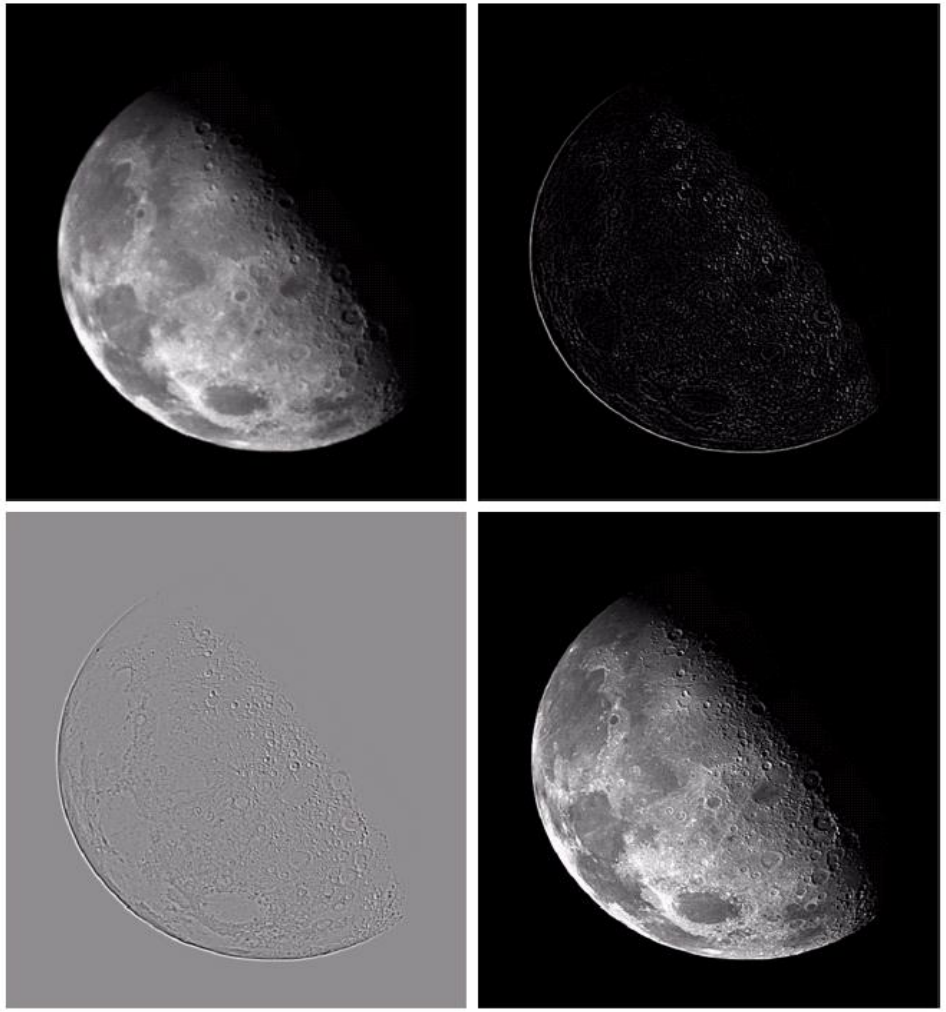
0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

➢ Sharpening Linear Filters

a b
c d

FIGURE 3.40
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)



From [Gonzalez & Woods]

➢ Sharpening Linear Filters

- **Image Sharpening Idea:**
  combining Laplacian with the image itself

  – Case 1: Center coefficient of the Laplacian mask is positive

$$g(x,y) = f(x,y) + \nabla^2 f(x,y)$$

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

  – Case 2: Center coefficient of the Laplacian mask is negative

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

➢ Sharpening Linear Filters

➢ Combined sharpening filters

$$g(x,y) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \otimes f(x,y) + \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \otimes f(x,y)$$

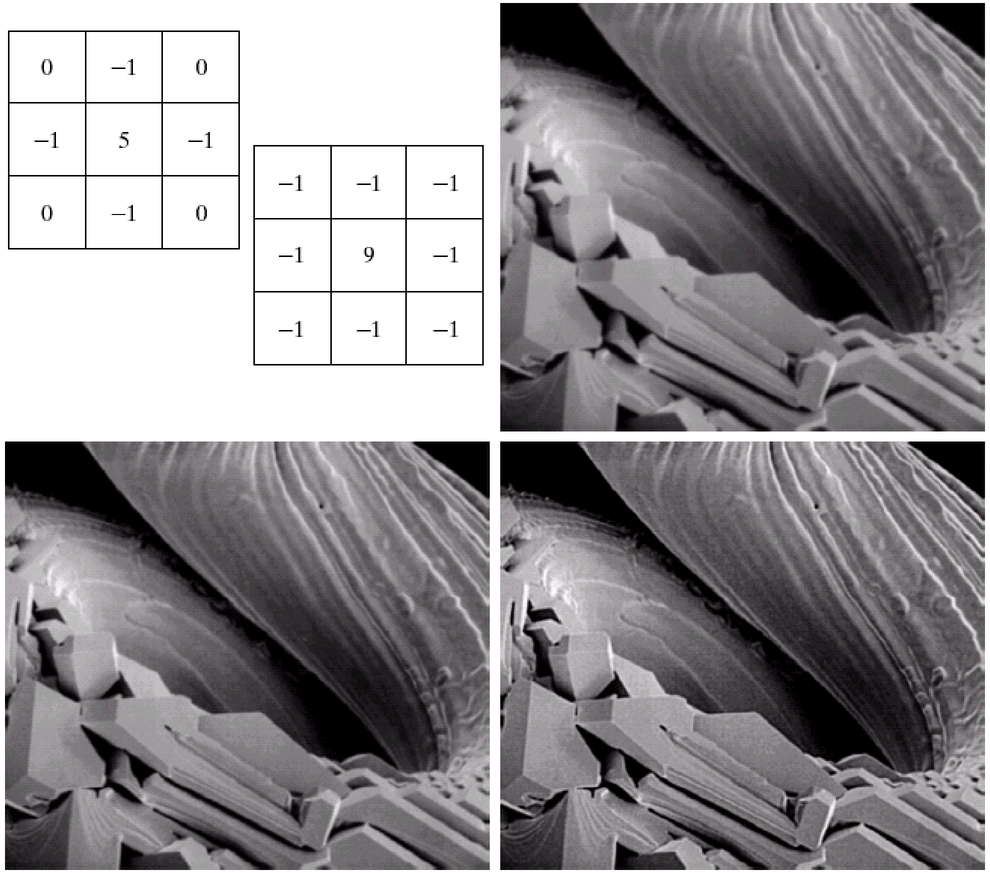
$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \otimes f(x,y)$$

$$g(x,y) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \otimes f(x,y) + \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \otimes f(x,y)$$

$$= \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix} \otimes f(x,y)$$

➢ Sharpening Linear Filters

From [Gonzalez & Woods]



0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)


Thank you